

# Studying possible CP-violating Higgs couplings through top-quark pair productions at muon colliders

**Zenrō Hioki**

*Institute of Theoretical Physics, University of Tokushima,  
Tokushima 770-8502, Japan  
E-mail: hioki@ias.tokushima-u.ac.jp*

**Takuya Konishi\***

*Graduate School of Human and Natural Environment Sciences, University of Tokushima,  
Tokushima 770-8502, Japan  
E-mail: konitaku747@ax5.mopera.ne.jp*

**Kazumasa Ohkuma**

*Department of Information Science, Fukui University of Technology,  
Fukui 910-8505, Japan  
E-mail: ohkuma@fukui-ut.ac.jp*

**ABSTRACT:** We study possible anomalous  $CP$ -violating Higgs couplings to  $\mu\bar{\mu}$  and  $t\bar{t}$  fully model-independent way through top-quark pair productions at muon colliders. Assuming additional non-standard neutral Higgs bosons, whose couplings with top-quark and muon are expressed in the most general covariant form, we carry out analyses of effects which they are expected to produce via  $CP$ -violating asymmetries and also the optimal-observable (OO) procedure under longitudinal and transverse muon polarizations. We find the measurement of the asymmetry for longitudinal beam polarization could be useful to catch some signal of  $CP$  violation, and an OO analysis might also be useful if we could reduce the number of unknown parameters with a help of other experiments and if the size of the parameters is at least  $O(1) \sim O(10)$ .

**KEYWORDS:** Heavy Quark Physics, Beyond Standard Model, CP violation, Higgs Physics.

---

\*Present affiliation: *NEC System Technologies, Ltd.*

---

## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. Framework</b>	<b>2</b>
<b>3. <math>CP</math>-violating asymmetries</b>	<b>4</b>
<b>4. Optimal-observable analysis</b>	<b>9</b>
<b>5. Summary</b>	<b>12</b>

---

## 1. Introduction

It is widely known that the standard model of the electroweak interaction (SM) has been so far quite successful in describing various phenomena below the electroweak scale with high precision. Its top-quark and Higgs-boson sectors are however still not fully-tested part of the model. If there exists any new physics beyond the SM within our reach, its effects will be likely to appear in those sectors. Therefore it is worth to look for experiments which allow for a comprehensive investigation of top-quark and Higgs-boson properties.

Anomalous top-quark interactions could be tested, for instance, at the  $e\bar{e}$  colliders in the International Linear Collider (ILC) project [1]. However it is not easy to study Higgs sector thereby. Muon colliders were proposed as an ideal machine to explore Higgs properties [2]. From a purely theoretical point of view, muon colliders are quite similar to  $e\bar{e}$  colliders, but the fact that a muon is much heavier than an electron could provide with a non-negligible difference in phenomenological studies of Higgs sector.

Indeed many authors have studied how to analyze Higgs-top interactions at muon colliders. Most of them focused on the resonance region, i.e., direct Higgs productions, and/or  $\mu\bar{\mu} \rightarrow \langle Higgs \rangle \rightarrow t\bar{t}$  in the framework of some specific models with multi Higgs doublets, like MSSM, and pointed out that a muon collider will be a useful tool to identify  $CP$  properties of Higgs scalars [2–4].

As a complementary work to them, we study in this article possible anomalous Higgs interactions with  $\mu\bar{\mu}$  and  $t\bar{t}$  in a fully model-independent way through  $\mu\bar{\mu} \rightarrow t\bar{t}$  processes. Our main purpose is to clarify to what extent we would be able to draw a general conclusion on those interactions without assuming any particular models at muon colliders in off-resonance region. In other words, we aim to study the possibility and limit of muon colliders for model-independent analyses of possible new physics in the top-quark and Higgs-boson sectors.

After describing our calculational framework in section 2, we compute  $CP$ -violating asymmetries for both longitudinal and transverse beam polarizations in section 3, where

based on the results we also discuss a detectability of the anomalous-coupling parameters. In section 4, we study whether the optimal-observable procedure is effective when we try to determine the anomalous parameters separately. Finally, a summary is given in section 5.

## 2. Framework

As mentioned in Introduction, we perform a model-independent analysis of possible non-standard Higgs interactions with top-quark/muon for longitudinal and transverse beam polarizations. Let us summarize our framework first which is the basis of our calculations. Throughout this paper, we express the standard-model Higgs as  $h$  and the non-standard neutral Higgs as  $H$ .

**Effective amplitude.** The invariant amplitude of  $\mu\bar{\mu} \rightarrow (\gamma, Z, h, H) \rightarrow t\bar{t}$  corresponding to figure 1 is given as follows:

$$\mathcal{M}(\mu\bar{\mu} \rightarrow t\bar{t}) = \mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_h + \mathcal{M}_H, \quad (2.1)$$

where  $\mathcal{M}_{\gamma,Z}$  are the standard  $\gamma$  and  $Z$  exchange terms

$$\mathcal{M}_\gamma = D_\gamma(s) \bar{u}(\mathbf{p}_t) \gamma^\alpha v(\mathbf{p}_{\bar{t}}) \cdot \bar{v}(\mathbf{p}_{\bar{\mu}}) \gamma_\alpha u(\mathbf{p}_\mu), \quad (2.2)$$

$$\mathcal{M}_Z = D_Z(s) \bar{u}(\mathbf{p}_t) \gamma^\alpha (A_t + B_t \gamma_5) v(\mathbf{p}_{\bar{t}}) \cdot \bar{v}(\mathbf{p}_{\bar{\mu}}) \gamma_\alpha (A_\mu + B_\mu \gamma_5) u(\mathbf{p}_\mu), \quad (2.3)$$

with

$$A_t = 1 - (8/3) \sin^2 \theta_W, \quad B_t = -1, \quad (2.4)$$

$$A_\mu = -1 + 4 \sin^2 \theta_W, \quad B_\mu = 1, \quad (2.5)$$

$D_{\gamma,Z}(s)$  being the propagators multiplied by the coupling constants

$$D_\gamma(s) \equiv -\frac{2}{3} e^2 \frac{1}{s}, \quad (2.6)$$

$$D_Z(s) \equiv \frac{g^2}{16 \cos^2 \theta_W} \frac{1}{s - M_Z^2}, \quad (2.7)$$

$e$ ,  $g$ ,  $\theta_W$  being the elementary charge, the SU(2) gauge coupling, the Weinberg angle respectively, and  $s \equiv (p_\mu + p_{\bar{\mu}})^2$ ,

$\mathcal{M}_h$  is the standard Higgs-boson exchange term

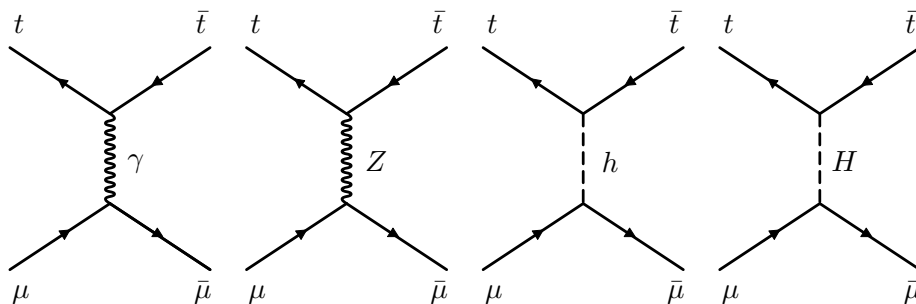
$$\mathcal{M}_h = D_h(s) \bar{u}(\mathbf{p}_t) v(\mathbf{p}_{\bar{t}}) \cdot \bar{v}(\mathbf{p}_{\bar{\mu}}) u(\mathbf{p}_\mu), \quad (2.8)$$

while  $\mathcal{M}_H$  is the non-standard Higgs exchange contribution, for which we assume the most general covariant form:

$$\mathcal{M}_H = D_H(s) \bar{u}(\mathbf{p}_t) (a_t + b_t \gamma_5) v(\mathbf{p}_{\bar{t}}) \cdot \bar{v}(\mathbf{p}_{\bar{\mu}}) (a_\mu + b_\mu \gamma_5) u(\mathbf{p}_\mu), \quad (2.9)$$

with

$$D_i(s) \equiv \frac{m_\mu m_t}{v^2} \frac{1}{m_i^2 - s - im_i \Gamma_i} \quad (2.10)$$



**Figure 1:** Feynman diagrams for  $\mu\bar{\mu} \rightarrow (\gamma, Z, h, H) \rightarrow t\bar{t}$ .

( $i = h, H$ ),  $\Gamma_i$  and  $v$  being the total decay width and the vacuum expectation value of the SM Higgs field. We treat  $a_{t,\mu}$  and  $b_{t,\mu}$  as complex numbers to take into account the possibility that they are form factors.

Readers may claim that we assume only one additional Higgs boson in spite of our statement that we perform a fully model-independent analysis. In fact, our frame can incorporate any number of Higgs exchange terms. It will be clear by re-expressing such an amplitude as

$$\begin{aligned}
 \mathcal{M}[\text{Non-SM Higgs}] &= \sum_{i=1}^N D_{H_i}(s) \bar{u}(\mathbf{p}_t)(a_t^i + b_t^i \gamma_5)v(\mathbf{p}_{\bar{t}}) \cdot \bar{v}(\mathbf{p}_{\bar{\mu}})(a_{\mu}^i + b_{\mu}^i \gamma_5)u(\mathbf{p}_{\mu}) \\
 &= \sum_i a_t^i a_{\mu}^i D_{H_i}(s) \bar{u}(\mathbf{p}_t)v(\mathbf{p}_{\bar{t}}) \cdot \bar{v}(\mathbf{p}_{\bar{\mu}})u(\mathbf{p}_{\mu}) \\
 &\quad + \sum_i a_t^i b_{\mu}^i D_{H_i}(s) \bar{u}(\mathbf{p}_t)v(\mathbf{p}_{\bar{t}}) \cdot \bar{v}(\mathbf{p}_{\bar{\mu}})\gamma_5 u(\mathbf{p}_{\mu}) \\
 &\quad + \sum_i b_t^i a_{\mu}^i D_{H_i}(s) \bar{u}(\mathbf{p}_t)\gamma_5 v(\mathbf{p}_{\bar{t}}) \cdot \bar{v}(\mathbf{p}_{\bar{\mu}})u(\mathbf{p}_{\mu}) \\
 &\quad + \sum_i b_t^i b_{\mu}^i D_{H_i}(s) \bar{u}(\mathbf{p}_t)\gamma_5 v(\mathbf{p}_{\bar{t}}) \cdot \bar{v}(\mathbf{p}_{\bar{\mu}})\gamma_5 u(\mathbf{p}_{\mu}).
 \end{aligned} \tag{2.11}$$

Thus, all the contributions can be packed into our parameters as follows:

$$a_t a_{\mu} = \sum_i a_t^i a_{\mu}^i D_{H_i}(s)/D_{H_1}(s), \tag{2.12}$$

$$a_t b_{\mu} = \sum_i a_t^i b_{\mu}^i D_{H_i}(s)/D_{H_1}(s), \tag{2.13}$$

$$b_t a_{\mu} = \sum_i b_t^i a_{\mu}^i D_{H_i}(s)/D_{H_1}(s), \tag{2.14}$$

$$b_t b_{\mu} = \sum_i b_t^i b_{\mu}^i D_{H_i}(s)/D_{H_1}(s). \tag{2.15}$$

**Beam polarization.** The beam polarization,  $P$ , along one axis (polarization axis) whose direction is defined by a unit vector  $\mathbf{s}$  is given by

$$P = \frac{\rho_{+s} - \rho_{-s}}{\rho_{+s} + \rho_{-s}}, \quad (2.16)$$

where  $\rho_{\pm s}$  is the number density of the particle in each beam whose spin component on this axis is  $\pm s$ . We can take into account this polarization by multiplying the spin vector  $s^\alpha$  in the projection operator  $u(\mathbf{p})\bar{u}(\mathbf{p})$  and  $v(\mathbf{p})\bar{v}(\mathbf{p})$  by  $P$ . That is, we are to use  $(0, P\mathbf{s})$  as the spin vector in its rest frame.

In the following, we choose the direction of  $\mathbf{p}_\mu$  as the  $z$  axis and express the azimuthal angle of  $\mathbf{s}$  as  $\phi$ . Then the degree of the longitudinal polarization is given by  $P_L = Ps_z$ , that of the transverse polarization by  $P_T = \sqrt{P^2 - P_L^2}$ , and consequently

$$(0, P\mathbf{s}) = (0, P_T \cos \phi, P_T \sin \phi, P_L). \quad (2.17)$$

The  $\mu$  and  $\bar{\mu}$  spin vectors in the  $\mu\bar{\mu}$  CM frame are obtained from (2.17) via appropriate Lorentz transformations as

$$s^\alpha = (P_L \gamma \beta, P_T \cos \phi, P_T \sin \phi, P_L \gamma), \quad (2.18)$$

$$\bar{s}^\alpha = (\bar{P}_L \gamma \beta, \bar{P}_T \cos \bar{\phi}, \bar{P}_T \sin \bar{\phi}, -\bar{P}_L \gamma), \quad (2.19)$$

where

$$\beta \equiv \sqrt{1 - 4m_\mu^2/s}, \quad \gamma \equiv 1/\sqrt{1 - \beta^2} \quad (2.20)$$

and the momenta of  $\mu$  and  $\bar{\mu}$  in this frame are

$$p^\alpha = \frac{1}{2}\sqrt{s}(1, 0, 0, \beta), \quad \bar{p}^\alpha = \frac{1}{2}\sqrt{s}(1, 0, 0, -\beta). \quad (2.21)$$

### 3. $CP$ -violating asymmetries

It is straightforward to calculate the cross section  $\sigma(\mu\bar{\mu} \rightarrow t\bar{t})$  starting from amplitude (2.1) as

$$\frac{d}{d\cos\theta}\sigma(\mu\bar{\mu} \rightarrow t\bar{t}) = \frac{1}{32\pi s} \frac{|\mathbf{p}_t|}{|\mathbf{p}_\mu|} |\mathcal{M}(\mu\bar{\mu} \rightarrow t\bar{t})|^2. \quad (3.1)$$

We perform this via FORM [5], but the analytical result is a bit too long to give here explicitly. Therefore we show in the following our results numerically. Throughout our analysis in this article, we take  $|P_L| = 1$  or  $|P_T| = 1$ . It may seem to be an extreme and unrealistic assumption, but we chose the polarization this way because our aim here is to know “to what extent” we could know about the anomalous interaction (2.9), i.e., we would like to study the possibility and limit of muon colliders for model-independent analyses of new physics beyond the standard model.

**Numerical results.** We study two  $CP$ -violating asymmetries  $A_L$  and  $A_T$ , the former of which is the one for longitudinal beam polarization

$$A_L = \frac{\sigma(++) - \sigma(--)}{\sigma(++) + \sigma(--)}, \quad (3.2)$$

and the latter is the one for transverse polarization

$$A_T = \frac{\sigma(\chi = \pi/2) - \sigma(\chi = -\pi/2)}{\sigma(\chi = \pi/2) + \sigma(\chi = -\pi/2)}, \quad (3.3)$$

where  $\sigma(\pm\pm)$  express the cross sections for  $P_L = \bar{P}_L = \pm 1$ , while  $\sigma(\chi = \pm\pi/2)$  are the ones for  $P_T = \bar{P}_T = 1$  with  $\chi \equiv \phi - \bar{\phi} = \pm\pi/2$ . We here chose  $|\chi|$  to be  $\pi/2$  since it maximizes the  $CP$ -violation effects (see, e.g., [4]).

Concerning the decay widths of  $h$  and  $H$ ,  $\Gamma_{h,H}$ , they are of course different quantities, but we use the same formula for  $\Gamma_H$  as  $\Gamma_h$  within the standard model [6] (see the later discussions). The other SM parameters are taken as follows:

$$\begin{aligned} \sin^2 \theta_W &= 0.23, & M_Z &= 91.187 \text{ GeV}, & v &= 246 \text{ GeV}, \\ m_t &= 174 \text{ GeV}, & m_\mu &= 105.658 \text{ MeV}, & m_h &= 150 \text{ GeV}. \end{aligned}$$

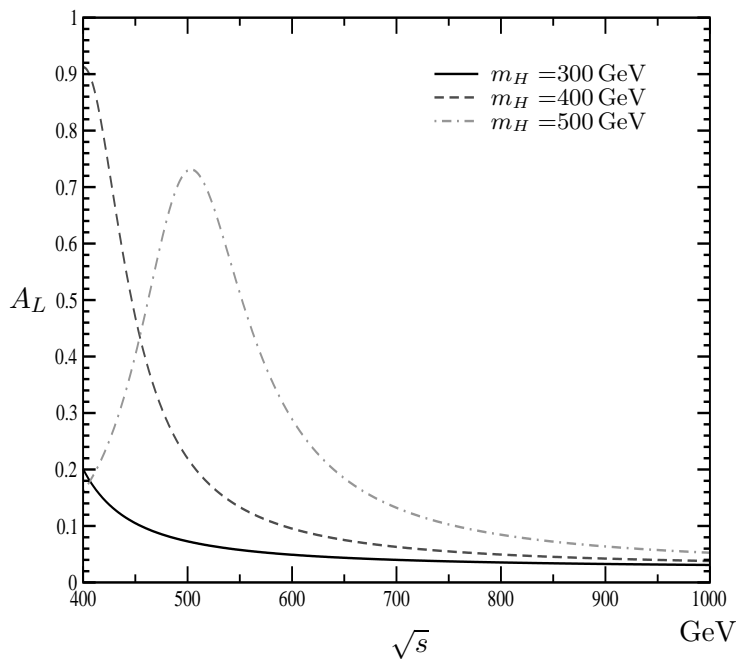
In figures 2 and 3 are presented  $A_L$  as functions of  $\sqrt{s}$  and  $m_H$ , while in figures 4 and 5 are given  $A_T$  in the same way for  $\text{Re } a_{t,\mu} = \text{Im } a_{t,\mu} = \text{Re } b_{t,\mu} = \text{Im } b_{t,\mu} = 0.2$  as an example to sketch a rough feature of these quantities. We find that the absolute value of  $A_L$  could be sizable, but that of  $A_T$  is very small.

It may seem strange that there appears such big difference between  $|A_L|$  and  $|A_T|$  though they are both  $CP$ -violating asymmetries computed with the same anomalous parameters. The reason is in their denominators. In the case of  $A_L$ , not only the numerator  $\sigma(++) - \sigma(--)$  but also the denominator receives little contribution from  $\gamma/Z$  exchange terms, while they can contribute to  $\sigma(\chi = \pm\pi/2)$  without being suppressed except in the difference  $\sigma(\chi = \pi/2) - \sigma(\chi = -\pi/2)$ . Indeed if we focus on the numerators alone, there is only small difference between  $A_L$  and  $A_T$ . For  $\sqrt{s} = 550 \text{ GeV}$  and  $m_H = 500 \text{ GeV}$  with the same anomalous couplings as in the figures, e.g.,

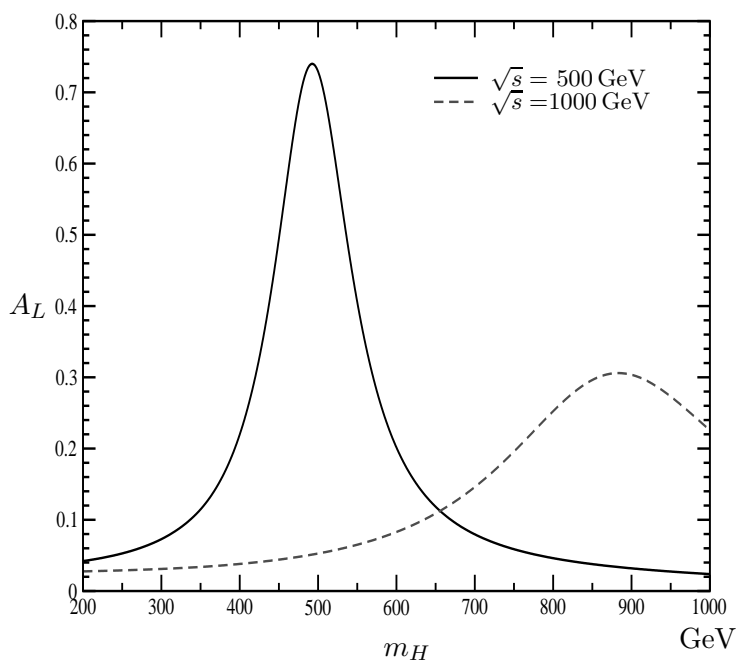
$$A_L : \quad \sigma(++) - \sigma(--)= 1.6 \times 10^{-2} \text{ fb}, \quad (3.4)$$

$$A_T : \quad \sigma(\chi = \pi/2) - \sigma(\chi = -\pi/2) = 4.8 \times 10^{-3} \text{ fb}. \quad (3.5)$$

It is obvious that the peaks in  $A_{L,T}$  are all due to the  $H$  propagator, but readers might wonder why  $A_T$  changes its sign while  $A_L$  not in the vicinity of  $s = M_H^2$ . Therefore, it would also be helpful to give a brief explanation here about those different behaviors of them. As mentioned above,  $\sigma(\chi = \pm\pi/2)$  receive sizable contributions from  $\gamma/Z$  exchange terms. This means the interference between the  $\gamma/Z$ -exchange and  $H$ -exchange terms, which is proportional to the  $H$  propagator, is important in  $A_T$ , and its sign changes thereby depending on whether  $s > M_H^2$  or  $s < M_H^2$ . On the other hand, this is no longer the case for  $A_L$  since the  $\gamma/Z$ -exchange terms are suppressed in both  $\sigma(++)$  and  $\sigma(--)$ . Therefore the sign of  $A_L$  is determined by the difference of  $\sigma(\pm\pm)$ , where the size of the amplitude

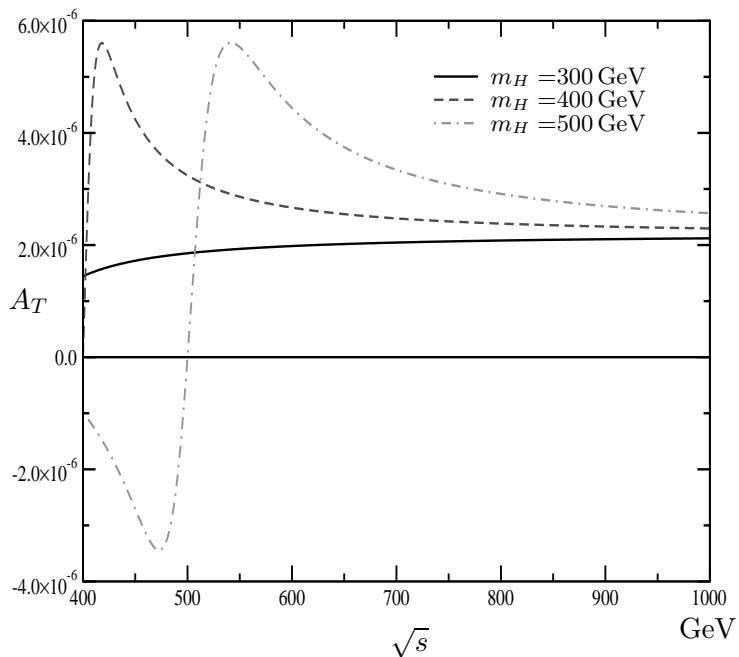


**Figure 2:**  $\sqrt{s}$  dependence of  $A_L$ .

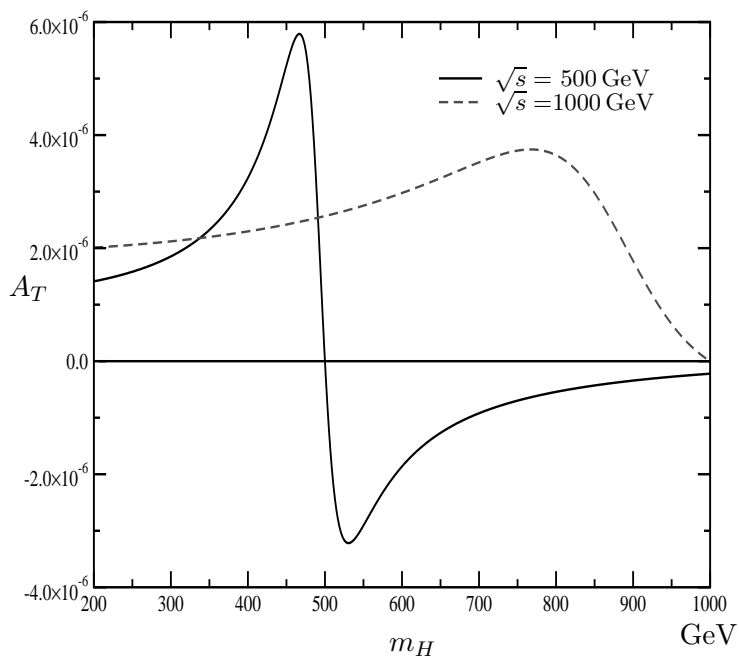


**Figure 3:**  $m_H$  dependence of  $A_L$ .

$\mathcal{M}_H$  itself is much more crucial. Here we chose  $a_\mu = b_\mu$  as an illustration, which makes  $\sigma(++)$  larger than  $\sigma(--)$  and leads to positive  $A_L$  as will be understood from eq. (4.6) on  $\mathcal{M}_H(++)$  and a similar calculation for  $\mathcal{M}_H(--)$ . This also tells us that different parameters, e.g.,  $a_\mu = -b_\mu$  could make  $A_L$  negative.



**Figure 4:**  $\sqrt{s}$  dependence of  $A_T$ .



**Figure 5:**  $m_H$  dependence of  $A_T$ .

**Detectability of the asymmetry.** Let us study the detectability of  $A_L$ , that is, the expected statistical precision in its measurement, which tells us how precisely we would be able to determine  $A_L$ . For instance if we take  $\sqrt{s} = m_H = 500$  GeV with  $\text{Re } a_{t,\mu} = \text{Re } b_{t,\mu} =$



$\sqrt{s}$ (GeV)	(a)			(b)			(c)		
	$A_L$	$N$	$N_{SD}$	$A_L$	$N$	$N_{SD}$	$A_L$	$N$	$N_{SD}$
450	0.08	7.6	0.2	0.40	11.7	1.5	0.73	25.7	5.4
480	0.19	9.4	0.6	0.64	21.1	3.8	0.88	61.8	14.4
500	0.26	10.5	0.9	0.73	28.7	5.7	0.92	91.6	21.8
520	0.23	10.1	0.7	0.69	25.2	4.8	0.90	77.8	18.2
550	0.12	8.8	0.4	0.51	15.9	2.4	0.81	40.9	8.9
600	0.05	7.7	0.1	0.29	10.4	1.0	0.63	19.8	3.6

**Table 1:**  $N_{SD}$  as a function of  $\sqrt{s}$  (with  $\epsilon = 1$  for simplicity) for  $\text{Re } a_{t,\mu} = \text{Re } b_{t,\mu} = \text{Im } a_{t,\mu} = \text{Im } b_{t,\mu} = 0.1$  (a), 0.2 (b), and 0.3 (c).

$\sqrt{s}$ (GeV)	$\Gamma_H = \Gamma_h(m_H)$	$\Gamma_H = 80$ GeV	$\Gamma_H = 100$ GeV
450	1.5	1.4	1.3
500	5.7	4.4	3.2
550	2.4	2.3	2.1
600	1.0	1.0	1.0

**Table 2:**  $N_{SD}$  as a function of  $\sqrt{s}$  for  $\Gamma_H = 80$  and 100 GeV, where  $\Gamma_H = \Gamma_h(m_H)$  means that  $\Gamma_H$  was computed with the SM formula ( $\Gamma_h(m_H) = 67.5$  GeV).

$\text{Im } a_{t,\mu} = \text{Im } b_{t,\mu} = 0.2$ ,  $A_L$  becomes 0.73, while the cross sections are  $\sigma(++) = 5.0 \times 10^{-2}$  fb and  $\sigma(-- ) = 7.8 \times 10^{-3}$  fb, leading to  $N \simeq 29\epsilon$  events for an integrated luminosity  $L = 500 \text{ fb}^{-1}$ , where we expressed the detection efficiency of  $t\bar{t}$  productions as  $\epsilon$ . They are combined to give the following statistical uncertainty:

$$\delta A_L = \sqrt{(1 - A_L^2)/N} = 0.68/\sqrt{\epsilon L} = 0.13/\sqrt{\epsilon}. \tag{3.6}$$

Consequently, the expected statistical significance  $N_{SD}$  is

$$N_{SD} \equiv |A_L|/\delta A_L = 5.7\sqrt{\epsilon}. \tag{3.7}$$

That is, we can confirm  $|A_L| \neq 0$  at  $5.7\sqrt{\epsilon}$  level. For example,  $N_{SD} = 4.0$  for  $\epsilon = 0.5$ . Here, assuming  $L = 500 \text{ fb}^{-1}$  may be a bit too optimistic, but we used this value considering that we aim to find the possibility and limit of the muon colliders as mentioned in the beginning of this section. It is easy to transform our numerical results for any other  $L$ .

We have given an example of  $N_{SD}$  for  $\sqrt{s} = m_H = 500$  GeV, but it is not general, so let us show the results for some other  $\sqrt{s}$  in table 1, changing also the parameters as  $\text{Re } a_{t,\mu} = \text{Re } b_{t,\mu} = \text{Im } a_{t,\mu} = \text{Im } b_{t,\mu} = 0.1, 0.2, 0.3$ . There we find that we would be able to observe some signal of  $CP$  violation as long as we are not too far from the  $H$  pole.

We used the SM formula for  $\Gamma_H$  as an appropriate approximation ( $\Gamma_H = 67.5$  GeV for  $m_H = 500$  GeV [6]), since we did not introduce any new light particles that can appear in the final state of  $h$  and  $H$  decays. Strictly speaking, however, a new mode like  $H \rightarrow hh$  might

	Re $a_t$	Re $a_\mu$	Re $b_t$	Re $b_\mu$	Im $a_t$	Im $a_\mu$	Im $b_t$	Im $b_\mu$
0.0	4.1	4.4	4.7	2.2	4.1	4.4	4.7	2.2
0.1	4.7	5.3	4.9	3.8	4.7	5.3	4.9	3.8
0.2	5.7	5.7	5.7	5.7	5.7	5.7	5.7	5.7
0.3	7.0	5.6	7.1	7.7	7.0	5.6	7.1	7.7

**Table 3:**  $N_{SD}$  as a function of each parameter for  $\sqrt{s} = 500$  GeV with the rest being fixed to be 0.2.

	Re $a_t$	Re $a_\mu$	Re $b_t$	Re $b_\mu$	Im $a_t$	Im $a_\mu$	Im $b_t$	Im $b_\mu$
0.0	1.1	2.0	2.0	0.4	2.3	1.5	1.9	1.8
0.1	1.7	2.3	2.1	1.3	2.3	1.9	2.0	2.1
0.2	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4
0.3	3.2	2.4	2.9	3.6	2.6	2.8	2.9	2.5

**Table 4:**  $N_{SD}$  as a function of each parameter for  $\sqrt{s} = 550$  GeV with the rest being fixed to be 0.2.

be possible for  $M_h = 150$  GeV and  $M_H = 500$  GeV. Instead of re-computing  $\Gamma_H$  including such new modes, which demands us to assume a concrete form of those couplings, we give  $N_{SD}$  for  $\Gamma_H = 80$  and  $100$  GeV with  $\text{Re } a_{t,\mu} = \text{Re } b_{t,\mu} = \text{Im } a_{t,\mu} = \text{Im } b_{t,\mu} = 0.2$  in table 2, which tells us that our conclusion would not be affected so much, especially in the off-resonance region.

**Parameter dependence of the asymmetry.** Measuring  $A_L$  would be quite interesting, but what  $A_L$  receives is of course one single combination of contributions from all the anomalous parameters. We are performing a model-independent analysis of possible new-physics effects, but once we get actual experimental data, our results are going to be applied for a realistic model building. If  $A_L$  does not depend on some parameters so much, it will be hard to test any models in which those parameters play a significant role. Therefore it must be important to see how  $A_L$  depends on each parameter.

Let us study how  $N_{SD}$  changes when we vary one parameter from 0.0 to 0.3. The results are given in tables 3 and 4, where  $N_{SD}$  are presented for one of the parameters = 0.0, 0.1, 0.2 and 0.3 with the others being fixed to be 0.2. There we observe that  $N_{SD}$  receives a contribution from every parameter though there are some differences among them, which indicates that any model will be testable through measuring  $A_L$ .

#### 4. Optimal-observable analysis

The optimal-observable technique [7] is a useful tool for estimating expected statistical uncertainties in various coupling measurements. Suppose we have a cross section

$$\frac{d\sigma}{d\phi} (\equiv \Sigma(\phi)) = \sum_i c_i f_i(\phi), \tag{4.1}$$

where  $f_i(\phi)$  are known functions of the final-state variables  $\phi$  and  $c_i$ 's are model-dependent coefficients. The goal is to determine the  $c_i$ 's. This can be done by using appropriate weighting functions  $w_i(\phi)$  such that  $\int w_i(\phi)\Sigma(\phi)d\phi = c_i$ . In general different choices for  $w_i(\phi)$  are possible, but there is a unique choice for which the resultant statistical error is minimized. Such functions are given by

$$w_i(\phi) = \sum_j X_{ij} f_j(\phi) / \Sigma(\phi), \tag{4.2}$$

where  $X_{ij}$  is the inverse matrix of  $\mathcal{M}_{ij}$  which is defined as

$$\mathcal{M}_{ij} \equiv \int \frac{f_i(\phi)f_j(\phi)}{\Sigma(\phi)} d\phi. \tag{4.3}$$

When we use these weighting functions, the statistical uncertainty of  $c_i$  is obtained as

$$\delta c_i = \sqrt{X_{ii} \sigma_T / N}, \tag{4.4}$$

where  $\sigma_T \equiv \int (d\sigma/d\phi)d\phi$  and  $N$  is the total number of events.

We study whether we could get more information of the anomalous parameters via this procedure. Here we focus on the longitudinal beam polarization, since we found that it is practically impossible to catch any new-physics signal for the transverse beam polarization even when we could fully use the total cross sections.

In order to apply this technique to our analysis, we need to express the angular distribution of the produced top quark in terms of the anomalous-coupling parameters like eq. (4.1). We have altogether eight independent parameters since we assumed the all couplings  $a_{t,\mu}$  and  $b_{t,\mu}$  to be complex. Although our aim is to perform an analysis as model-independently as possible, it will be too complicated to treat them all equally. Therefore we here assume that the size of the imaginary part of each parameter is much smaller than that of its real part. Since the imaginary part of parameters (form factors) is often produced through higher order loop corrections in an underlying theory, this assumption is not unreasonable. We also drop the terms quartic in the anomalous parameters.

With this reduced parameter set and assumption, the top-quark angular distribution should be represented as

$$\frac{d}{d \cos \theta} \sigma_{++}(\mu\bar{\mu} \rightarrow t\bar{t}) = f_{\text{SM}}(\theta) + c_{aa}f_{aa}(\theta) + c_{ab}f_{ab}(\theta) + c_{ba}f_{ba}(\theta) + c_{bb}f_{bb}(\theta), \tag{4.5}$$

where  $f_{\text{SM}}(\theta)$  expresses the SM contribution,

$$\begin{aligned} c_{aa} &\equiv (\text{Re } a_t)(\text{Re } a_\mu), & c_{ab} &\equiv (\text{Re } a_t)(\text{Re } b_\mu), \\ c_{ba} &\equiv (\text{Re } b_t)(\text{Re } a_\mu), & c_{bb} &\equiv (\text{Re } b_t)(\text{Re } b_\mu), \end{aligned}$$

and  $f_{ij}(\theta)$  ( $i, j = a, b$ ) are all independent of each other. If we reverse the signs of  $c_{ab}$  and  $c_{ba}$ , we get  $d\sigma_{--}(\mu\bar{\mu} \rightarrow t\bar{t})$ .

Practically, however,  $f_{ia}(\theta)$  and  $f_{ib}(\theta)$  become equivalent in the limit of  $m_\mu \rightarrow 0$ . We can see this as follows: In calculating  $d\sigma_{++}$ , the muon-spinor part becomes

$$\begin{aligned} \bar{v}_+(\mathbf{p}_{\bar{\mu}})(a_\mu + b_\mu \gamma_5)u_+(\mathbf{p}_\mu) &\simeq \bar{v}_+(\mathbf{p}_{\bar{\mu}})(a_\mu + b_\mu \gamma_5) \frac{1 + \gamma_5}{2} u(\mathbf{p}_\mu) \\ &= (a_\mu + b_\mu) \bar{v}_+(\mathbf{p}_{\bar{\mu}})u_+(\mathbf{p}_\mu) \end{aligned} \tag{4.6}$$

in the limit. That is,  $a_\mu$  and  $b_\mu$  contribute almost equally to  $d\sigma_{++}$ , which leads to

$$f_{aa}(\theta) \simeq f_{ab}(\theta), \quad f_{ba}(\theta) \simeq f_{bb}(\theta).$$

Since we keep  $m_\mu$  finite, it is in principle possible to perform an analysis treating all  $f_{ij}(\theta)$  as independent functions, but it is clear that we end up having very poor precision thereby [8]. Therefore we neglect their differences from the beginning and start from

$$\frac{d}{d \cos \theta} \sigma_{++}(\mu\bar{\mu} \rightarrow t\bar{t}) \simeq f_1(\theta) + c_a f_2(\theta) + c_b f_3(\theta), \quad (4.7)$$

where  $c_a \equiv c_{aa} + c_{ab}$ ,  $c_b \equiv c_{ba} + c_{bb}$ ,  $f_1(\theta) = f_{\text{SM}}(\theta)$ ,  $f_2(\theta) = f_{aa}(\theta) \simeq f_{ba}(\theta)$  and  $f_3(\theta) = f_{ba}(\theta) \simeq f_{bb}(\theta)$ .

Using these functions we obtain the following results as the matrix (4.3) for  $\sqrt{s} = 550$  GeV and  $m_H = 500$  GeV:

$$\begin{aligned} M_{11} &= 7.75 \cdot 10^{-3}, & M_{12} &= 5.80 \cdot 10^{-2}, & M_{13} &= -2.38 \cdot 10^{-3}, \\ M_{22} &= 4.35 \cdot 10^{-1}, & M_{23} &= -1.79 \cdot 10^{-2}, & M_{33} &= 7.37 \cdot 10^{-4}, \end{aligned} \quad (4.8)$$

where we used  $f_1(\theta)$  for  $\Sigma(\phi)$  in eq. (4.3). We then compute the (2, 2) and (3, 3) elements of the inverse matrix of  $M$ :

$$X_{22} = 3.68 \cdot 10^6, \quad X_{33} = 5.38 \cdot 10^8.$$

This means the expected statistical uncertainty in  $c_{a,b}$  measurements are

$$\delta c_a = 1.92 \cdot 10^3 / \sqrt{L}, \quad \delta c_b = 2.32 \cdot 10^4 / \sqrt{L}. \quad (4.9)$$

This tells us that we need  $L = 3.7 \cdot 10^6 \text{ fb}^{-1}$  for achieving  $\delta c_a = 1$  and  $L = 5.4 \cdot 10^8 \text{ fb}^{-1}$  for  $\delta c_b = 1$ , which are both far beyond our reach!

We then assume one of the parameters is determined in some other experiments in order to look for realistic solutions. First, if  $c_a$  was unknown (i.e., if  $c_b$  was measured elsewhere), the corresponding precision becomes  $\delta c_a = 44.5 / \sqrt{L}$ , i.e.,  $\delta c_a = 1.99$  for  $L = 500 \text{ fb}^{-1}$ . We give also results for some other  $\sqrt{s}$  in table 5. Conversely, if  $c_b$  is undetermined (i.e., only  $c_a$  is known), we have  $\delta c_b = 539 / \sqrt{L}$ , i.e.,  $\delta c_b = 24.1$  for  $L = 500 \text{ fb}^{-1}$ . Some other results are in table 6.

Therefore, if the size of  $c_a$  is  $O(1)$ , there is some hope to catch new-physics signal thereby. On the other hand,  $|c_b|$  is required to be at least  $O(10)$ . Note that it is never unrealistic to assume  $|c_{a,b}|$  to be  $O(1) \sim O(10)$  as is known in various models with two (or multi) Higgs-doublets (see, e.g., [9] and the references therein).

What we could know via  $A_L$  measurements is only on  $CP$  violation, while  $c_{a,b}$  are both combinations of  $CP$ -conserving and  $CP$ -violating parameters. Therefore those two approaches could work complementarily to each other.

$\sqrt{s}$ (GeV)	$\delta c_a$
400	$63.7/\sqrt{L}$
450	$41.1/\sqrt{L}$
480	$44.7/\sqrt{L}$
520	$44.7/\sqrt{L}$
580	$61.4/\sqrt{L}$
600	$75.6/\sqrt{L}$

**Table 5:** Expected precision of  $c_a$  determination for  $m_H = 500$  GeV.

$\sqrt{s}$ (GeV)	$\delta c_b$
400	$168/\sqrt{L}$
450	$227/\sqrt{L}$
480	$329/\sqrt{L}$
520	$447/\sqrt{L}$
580	$878/\sqrt{L}$
600	$1197/\sqrt{L}$

**Table 6:** Expected precision of  $c_b$  determination for  $m_H = 500$  GeV.

## 5. Summary

We have carried out a model-independent analysis of possible non-standard Higgs interactions with  $t\bar{t}$  and  $\mu\bar{\mu}$  through top-quark pair productions at future muon colliders. As was pointed out in refs. [2–4], the muon colliders are quite useful for studying the Higgs sector around the resonance. Considering those preceding studies, our main purpose here was to see if we could also draw any useful information in the off-resonance region without depending on any specific models.

Starting from the most general covariant amplitude, we computed two  $CP$ -violating asymmetries for longitudinal and transverse beam polarizations in order to see if we could get any signal of new-physics which breaks  $CP$  symmetry, and also studied whether we could determine the non-standard-coupling parameters separately through the optimal-observable (OO) procedure as a more detailed analysis.

We found that the longitudinal  $CP$ -violating asymmetry  $A_L$  would be sizable, while the transverse asymmetry  $A_T$  is too small to be a meaningful observable. We then estimated the detectability of  $A_L$  and showed that we would be able to observe some signal of  $CP$  violation as long as we are not too far from the  $H$  pole. We also studied there in some detail how  $A_L$  depends on each parameter, and found that we have no parameter that contribute little, although there are some differences among the parameters.

On the other hand, more detailed analyses via the OO procedure seem challenging. However, if we could reduce the number of unknown parameters with a help of other experiments, and if the size of the parameters is at least  $O(1) \sim O(10)$ , we might be able to get some meaningful information thereby. Readers may claim that the use of the

asymmetry  $A_L$  is enough when we have only one unknown parameter, but this is not necessarily true. What we could draw from  $A_L$  is information on pure  $CP$  violation, while we could also know something about  $CP$ -conserving part through an  $OO$  analysis.

In our approach, we need the total cross section of  $\mu\bar{\mu} \rightarrow t\bar{t}$  and the angular distribution of the final top quark, for which we only have to reconstruct the top-quark jet axis. If we further try to study, e.g., the final lepton distributions in the top-quark decays, we could get additional information on possible anomalous  $tbW$  coupling, however in that case we would suffer from another suppression factor, i.e., the branching ratio of the top-quark semileptonic decay. Therefore it will be more advantageous to use the top-quark distribution as a whole when performing off-resonance analyses at muon colliders.

## Acknowledgments

We are grateful to Federico von der Pahlen for giving us a valuable comment about the integrated luminosity we used in the text and also pointing out some typos in the manuscript. This work is supported in part by the Grant-in-Aid for Scientific Research No.13135219 and No.16540258 from the Japan Society for the Promotion of Science, and the Grant-in-Aid for Young Scientists No. 17740157 from the Ministry of Education, Culture, Sports, Science and Technology of Japan. The algebraic calculations using FORM were carried out on the computer system at Yukawa Institute for Theoretical Physics (YITP), Kyoto University.

## References

- [1] See ILC web site <http://www.linearcollider.org/cms/>.
- [2] V.D. Barger, M.S. Berger, J.F. Gunion and T. Han, *s-channel Higgs boson production at a muon-muon collider*, *Phys. Rev. Lett.* **75** (1995) 1462 [[hep-ph/9504330](#)]; *Higgs boson physics in the s-channel at  $\mu^+\mu^-$  colliders*, *Phys. Rept.* **286** (1997) 1 [[hep-ph/9602415](#)].
- [3] B. Grzadkowski and J.F. Gunion, *Using decay angle correlations to detect CP violation in the neutral Higgs sector*, *Phys. Lett.* **B 350** (1995) 218 [[hep-ph/9501339](#)];  
D. Atwood and A. Soni, *Neutral-Higgs-boson CP violation at  $\mu^+\mu^-$  colliders*, *Phys. Rev.* **D 52** (1995) 6271 [[hep-ph/9505233](#)];  
S.Y. Choi and J.S. Lee, *s-channel production of MSSM Higgs bosons at a muon collider with explicit CP violation*, *Phys. Rev.* **D 61** (2000) 111702 [[hep-ph/9909315](#)];  
E. Asakawa, A. Sugamoto and I. Watanabe, *Production of CP-even and CP-odd Higgs bosons at muon colliders*, *Eur. Phys. J.* **C 17** (2000) 279;  
E. Asakawa, S.Y. Choi and J.S. Lee, *Probing the MSSM Higgs boson sector with explicit CP violation through third generation fermion pair production at muon colliders*, *Phys. Rev.* **D 63** (2001) 015012 [[hep-ph/0005118](#)];  
see also A. Pilaftsis, *Resonant CP-violation at  $\mu^+\mu^-$  colliders*, *Phys. Rev. Lett.* **77** (1996) 4996 [[hep-ph/9603328](#)].
- [4] B. Grzadkowski, J.F. Gunion and J. Pliszka, *How valuable is polarization at a muon collider? A test case: determining the CP nature of a Higgs boson*, *Nucl. Phys.* **B 583** (2000) 49 [[hep-ph/0003091](#)].

- [5] J.A.M. Vermaseren, *Symbolic manipulation with FORM*, version 2, Tutorial and Reference Manual, Computer Algebra Nederland, Amsterdam (1991).
- [6] A. Djouadi, J. Kalinowski and M. Spira, *HDECAY: a program for Higgs boson decays in the standard model and its supersymmetric extension*, *Comput. Phys. Commun.* **108** (1998) 56 [[hep-ph/9704448](#)].
- [7] D. Atwood and A. Soni, *Analysis for magnetic moment and electric dipole moment form factors of the top quark via  $e^+e^- \rightarrow t\bar{t}$* , *Phys. Rev.* **D 45** (1992) 2405;  
M. Davier, L. Duflot, F. Le Diberder and A. Rouge, *The optimal method for the measurement of tau polarization*, *Phys. Lett.* **B 306** (1993) 411;  
M. Diehl and O. Nachtmann, *Optimal observables for the measurement of three gauge boson couplings in  $e^+e^- \rightarrow W^+W^-$* , *Z. Physik* **C 62** (1994) 397;  
J.F. Gunion, B. Grzadkowski and X.-G. He, *Determining the  $t\bar{t}$  and ZZ couplings of a neutral Higgs boson of arbitrary CP nature at the next linear collider*, *Phys. Rev. Lett.* **77** (1996) 5172 [[hep-ph/9605326](#)].
- [8] B. Grzadkowski, Z. Hioki, K. Ohkuma and J. Wudka, *Probing anomalous top-quark couplings induced by dim.6 operators at photon colliders*, *Nucl. Phys.* **B 689** (2004) 108 [[hep-ph/0310159](#)]; *Optimal-observable analysis of possible new physics using the b-quark in  $\gamma\gamma \rightarrow t\bar{t} \rightarrow bX$* , *Phys. Lett.* **B 593** (2004) 189 [[hep-ph/0403174](#)]; *Optimal beam polarizations for new-physics search through  $\gamma\gamma \rightarrow t\bar{t} \rightarrow lX/bX$* , *JHEP* **11** (2005) 029 [[hep-ph/0508183](#)].
- [9] J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, *The Higgs hunter's guide*, Perseus Publishing, Cambridge, Massachusetts.